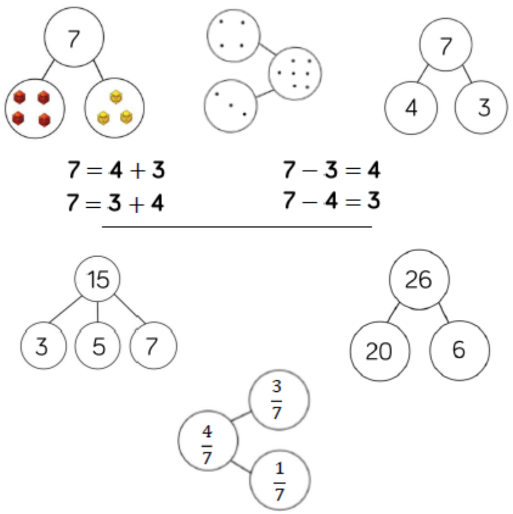
Thinking Academically

Part-Whole Model



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

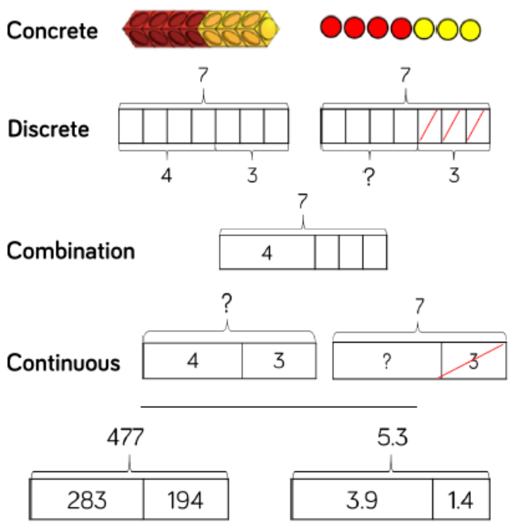
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

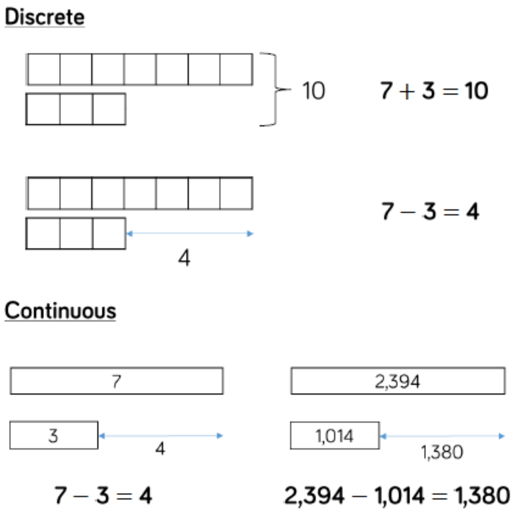
Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)



Benefits

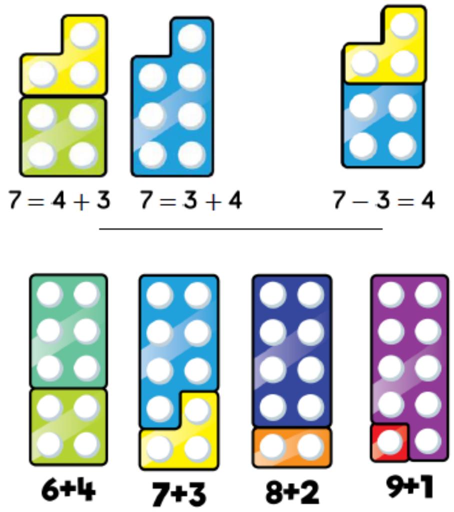
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes



Benefits

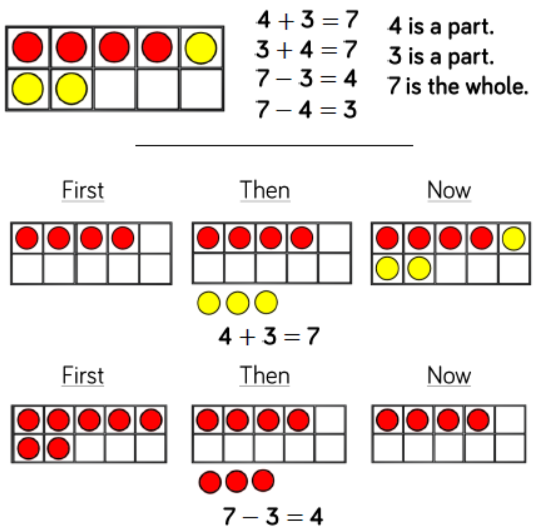
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Ten Frames (within 10)



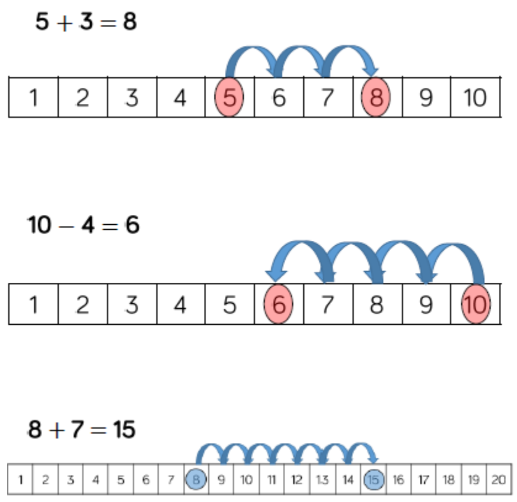
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Number Tracks



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

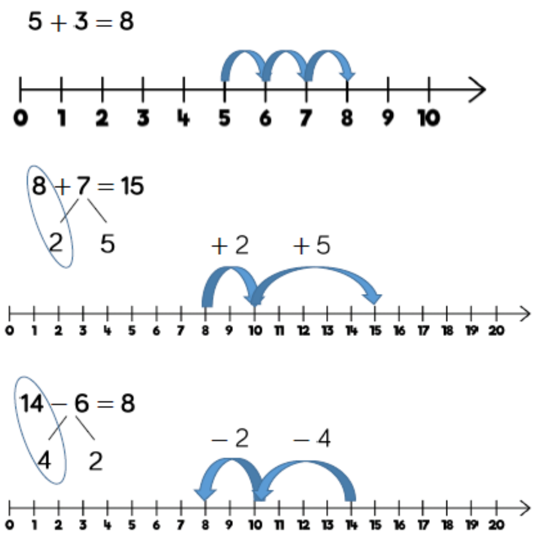
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)



Benefits

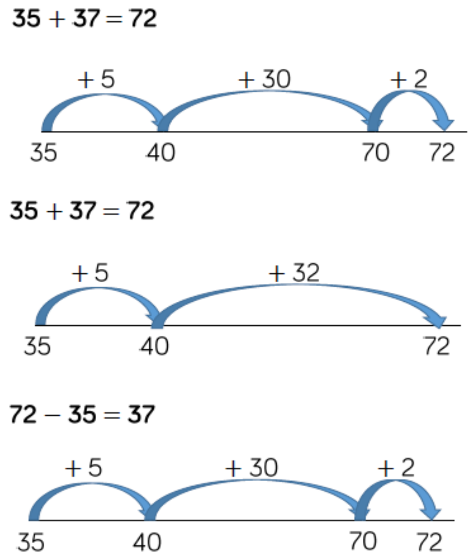
Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)



Benefits

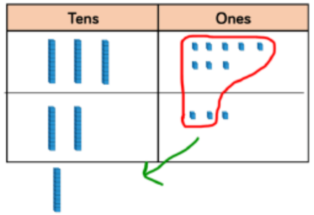
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

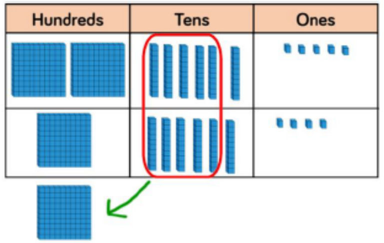
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10/Dienes (addition)



38
+ 23
61
1



265
+ 164
429
1

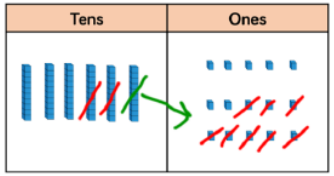
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

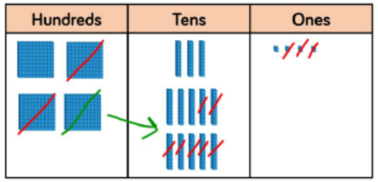
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes (subtraction)



5 1
~~6~~5
– 28
37



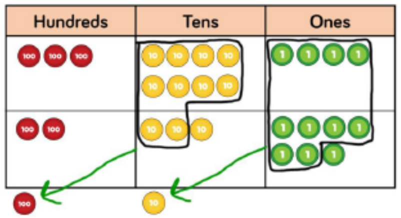
3 1
~~4~~35
– 273
262

Benefits

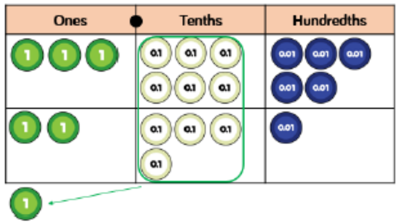
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



384
+ 237
621
1 1



3.65
+ 2.41
6.06
1

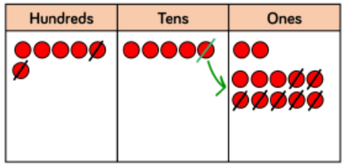
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

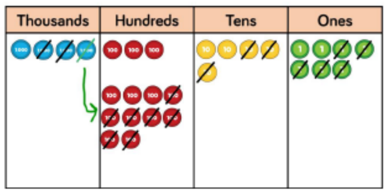
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)



4 1
6~~5~~2
– 207
445



3 1
~~4~~357
– 2735
1622

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

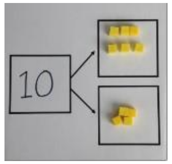
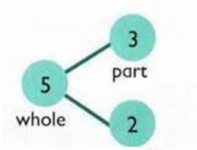
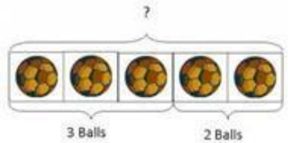
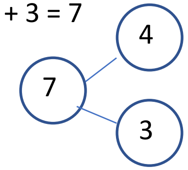
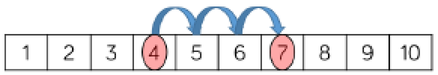
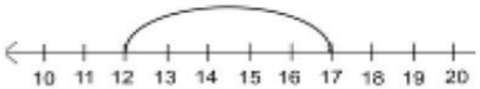
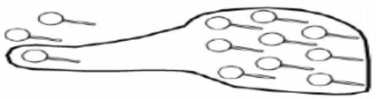
When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

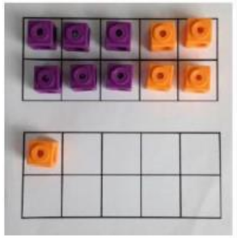
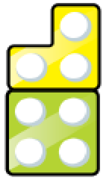
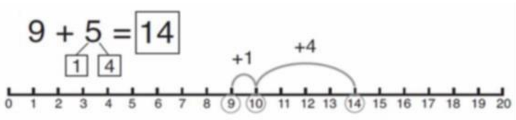
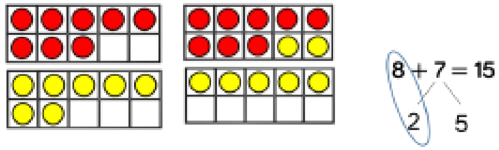
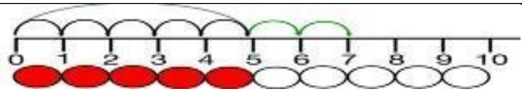
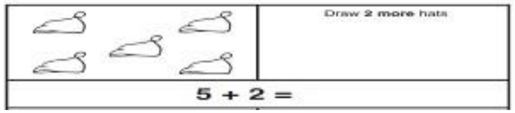
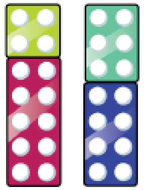
'Thinking Academically' Calculation Policy

Child First – Aspire – Challenge – Achieve

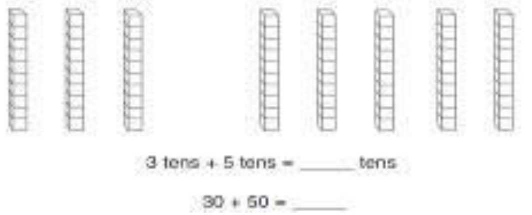
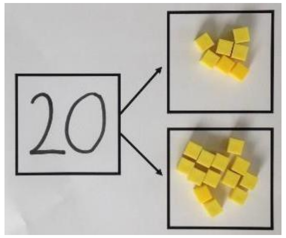
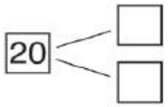
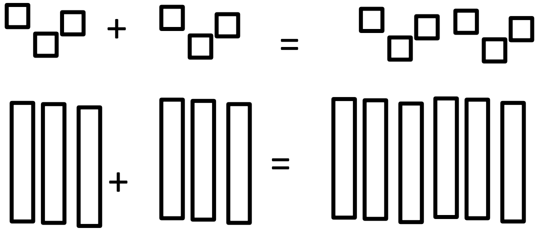
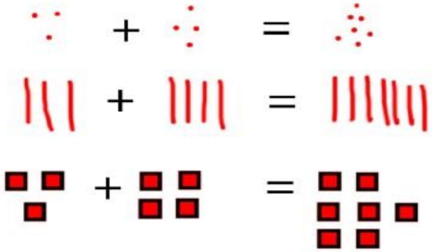
Thinking Academically

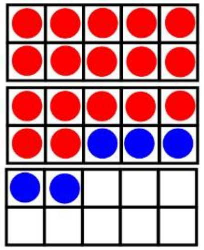
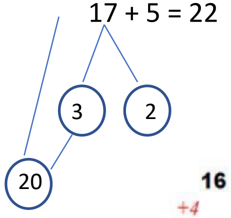
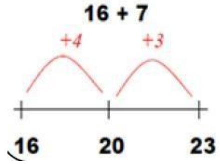
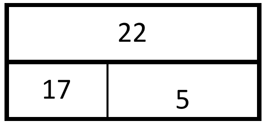
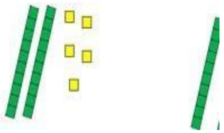
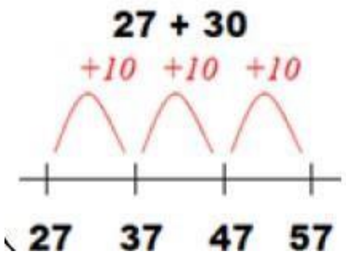
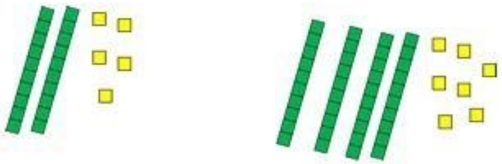
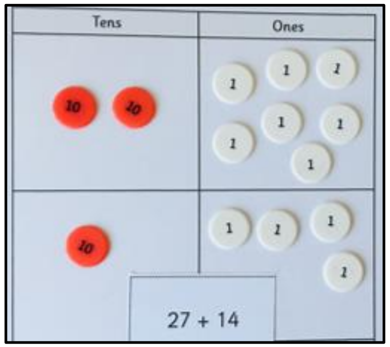
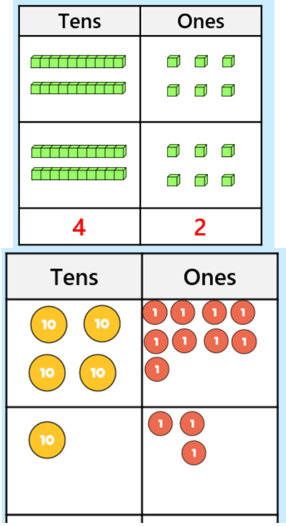
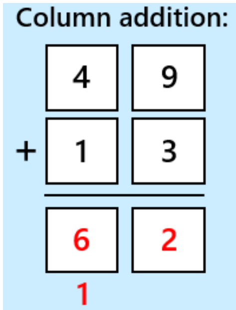
Year 1 Addition

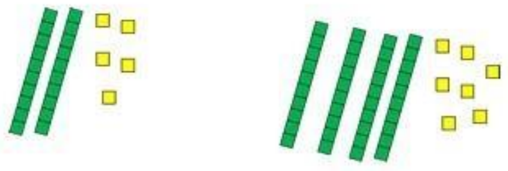
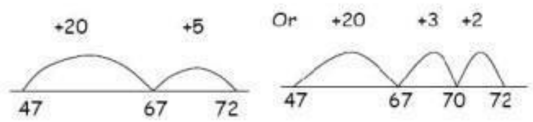
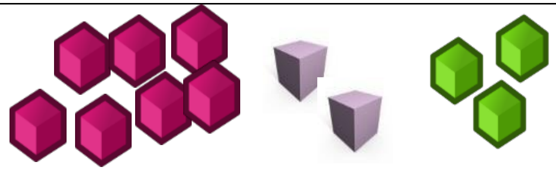
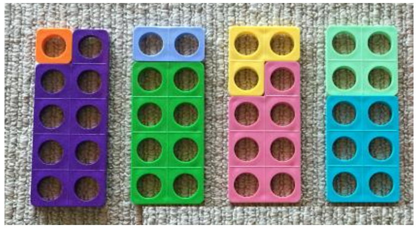
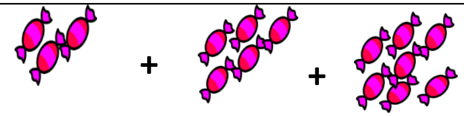
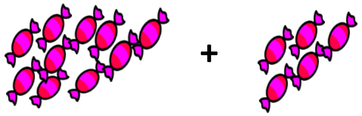
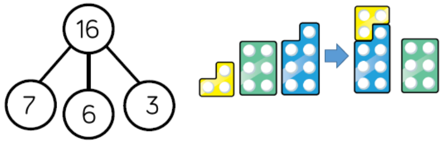
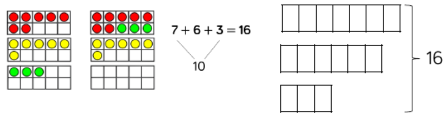
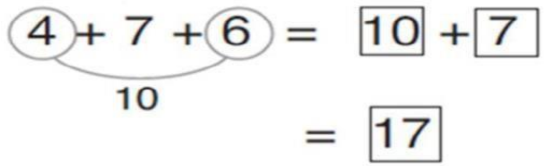
Objective and Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	   Use part whole model. Use cubes to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7  10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	  12 + 5 = 17 Start at the larger number on the number line/number track and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	 6 + 5 = 11	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

	 Start with the bigger number and use the smaller number to make 10. Use ten frames. 	 	
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	  	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

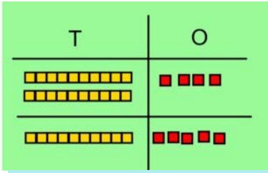
Year 2 Addition

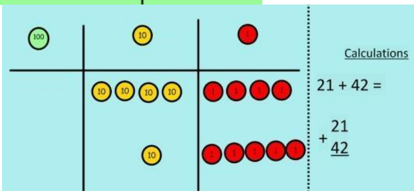
Objective and Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts **Part whole**	 Children explore ways of making numbers within 20	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 **Leads to** 30 + 40 + 70 **Leads to** 300 + 400 = 700
Bar model	 3 + 4 = 7	7 + 3 = 10 	7 + 3 = 10 7 3 10

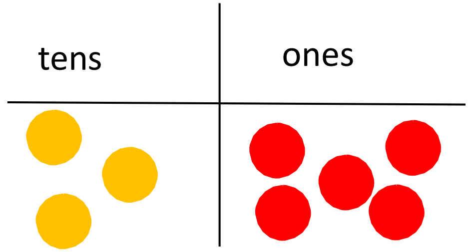
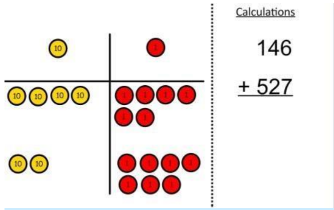
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make ‘magic ten’ Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part whole and number line to model  	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Using the column method	 		T O 2 6 1 6 4 2 1 

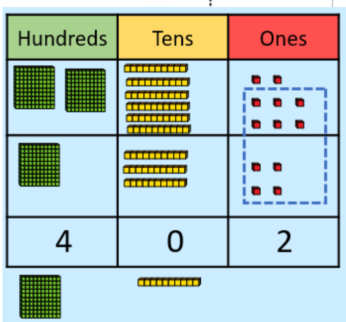
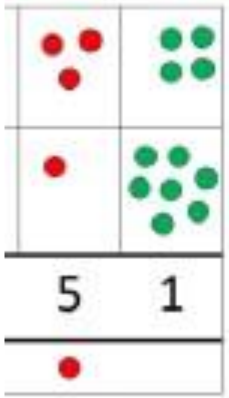
Add two 2-digit numbers	 Model using dienes, place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	25 + 47 = **72** 20 + 5 40 + 7 20 + 40 = **60** 5 + 7 = **12** **60** + **12** = **72**
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit 	 Regroup and draw representation.  = 15  7 + 6 + 3 = 16 	 Combine the two numbers that make/ bridge ten then add on the third. Children should be encouraged to look for number bonds to 10 or doubles to add more efficiently. Emphasis on commutativity.

<table>
<tr><th colspan="4">Year 3 Addition</th></tr>
<tr><th>Objective and Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column Addition—no regrouping (friendly numbers)

Add two or three 2 or 3- digit numbers.</td>
<td>

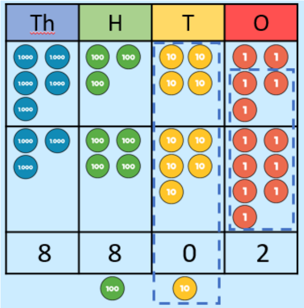
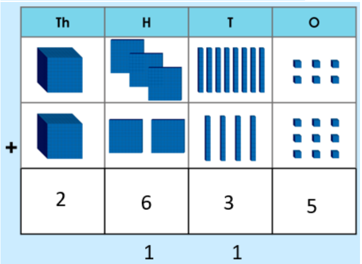
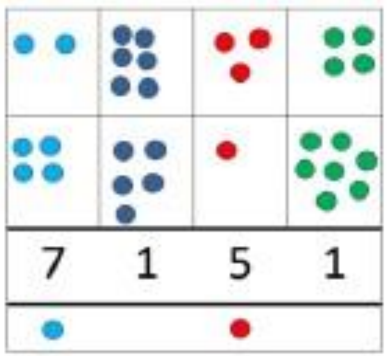
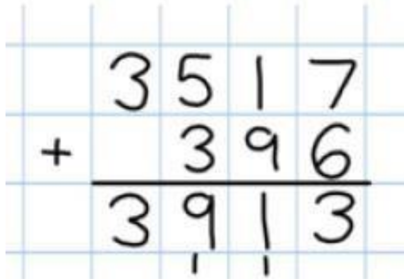
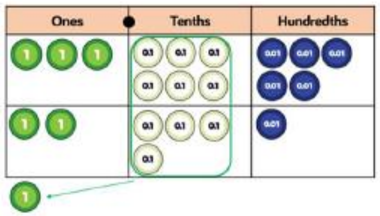
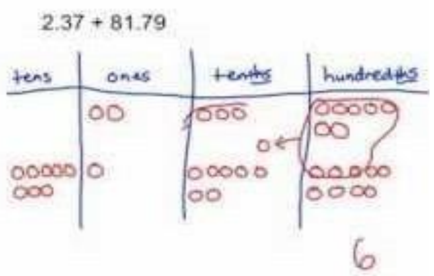
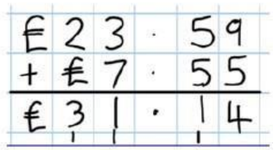
Model using Dienes or place value counters


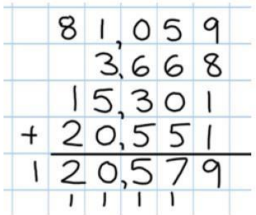
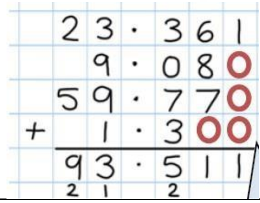
Add together the ones first, then the tens.</td>
<td>Children move to drawing the counters or dienes using a tens and one frame

</td>
<td>$$\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$$
Add the ones first, then the tens, then the hundreds.</td>
</tr>
<tr>
<td>Column Addition with regrouping.</td>
<td>Exchange ten ones for a ten.
Model using dienes or place value counters.



</td>
<td>

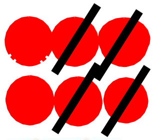
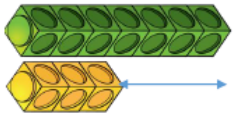
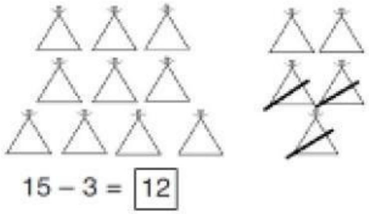
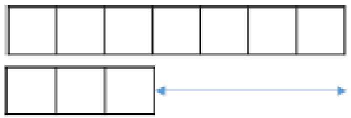
Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</td>
<td>$$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 1\,1 \end{array}$$
<table>
<tr><th></th><th>H</th><th>T</th><th>O</th></tr>
<tr><td></td><td>2</td><td>6</td><td>8</td></tr>
<tr><td>+</td><td>1</td><td>3</td><td>4</td></tr>
<tr><td></td><td>4</td><td>0</td><td>2</td></tr>
<tr><td></td><td>1</td><td>1</td><td></td></tr>
</table></td>
</tr>
</table>

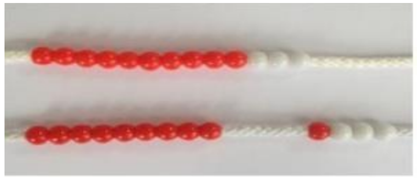
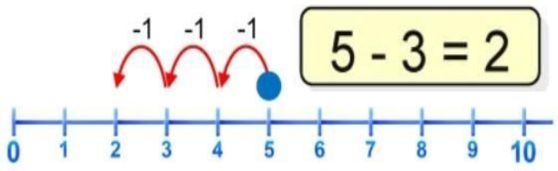
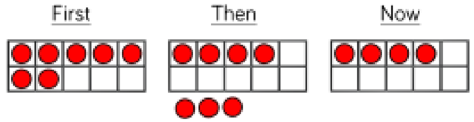
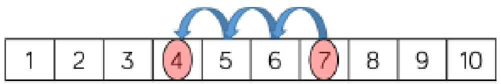
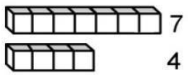
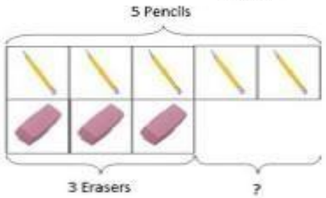
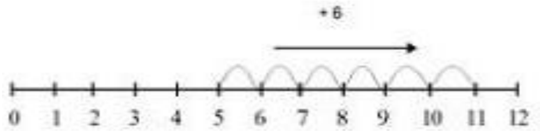
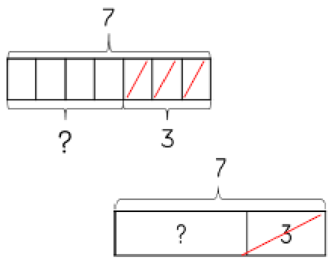
Year 4 to Year 6 Addition

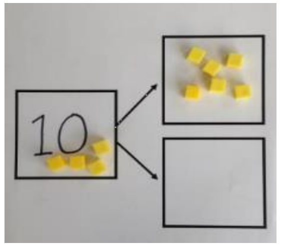
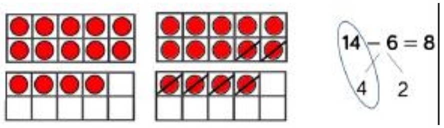
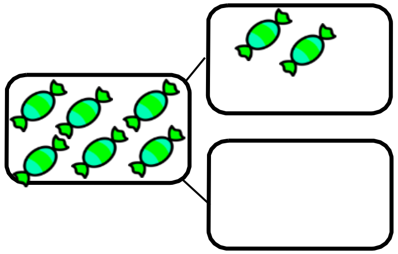
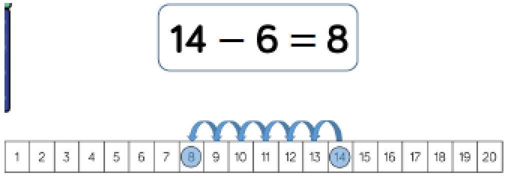
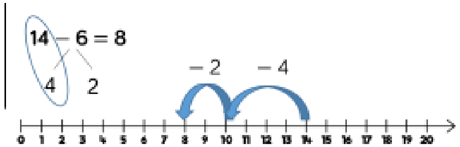
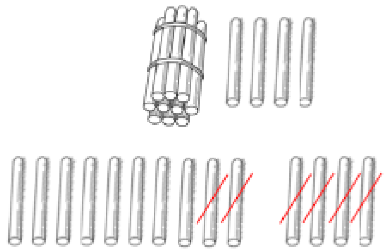
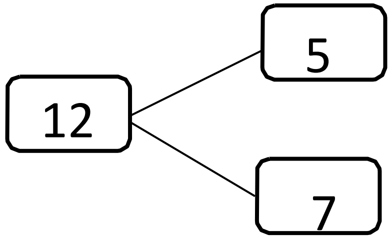
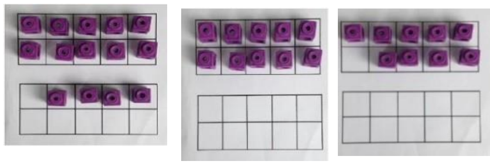
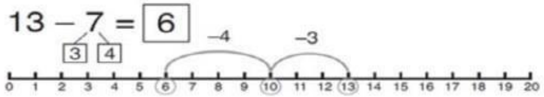
Objective and Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.   	 Draw representations (dienes of pv coutners) using pv grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. **Add decimals with 2 decimal places, including money.**	 Introduce decimal place value counters and model exchange for addition.	 	72.8 + 54.6 127.4 1 1 

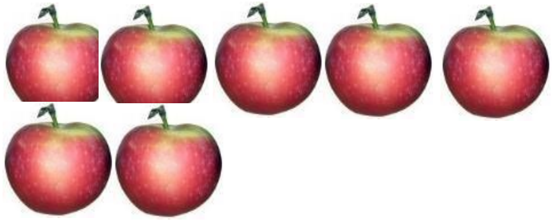
Y6—add several numbers of increasing complexity **Including adding money, measure and decimals with different numbers of decimal points.**	As Year 5	As Year 5	  Insert zeros for place holders.

Year 1 Subtraction

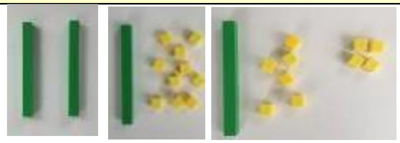
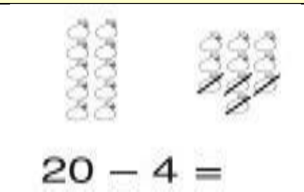
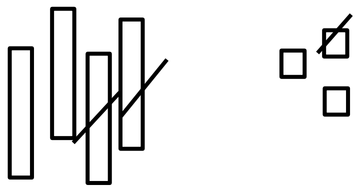
Objective and Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, to show how objects, counters, cubes etc can be taken away.  6—4 = 2  4—2 = 2 	 Cross out drawn objects to show what has been taken away. 	7—4 = 3 16—9 = 7 **7 – 3 = 4**

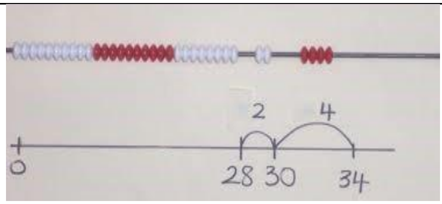
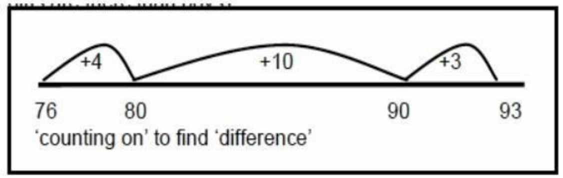
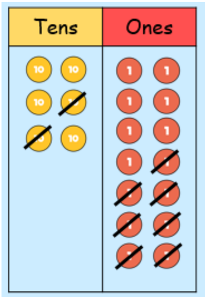
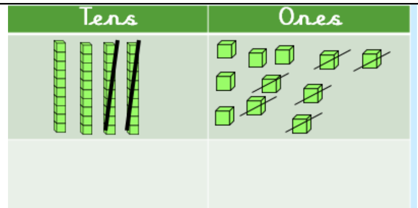
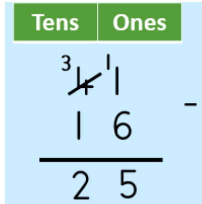
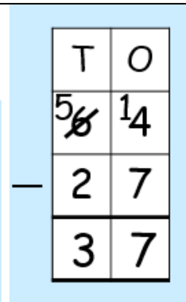
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.  	Put 13 in your head, count back 4. What number are you at?
Find the difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent bar model.	Count on using a number line to find the difference.  	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

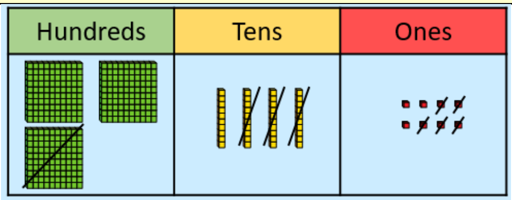
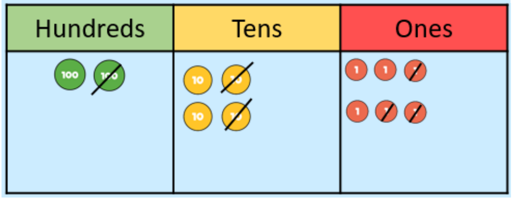
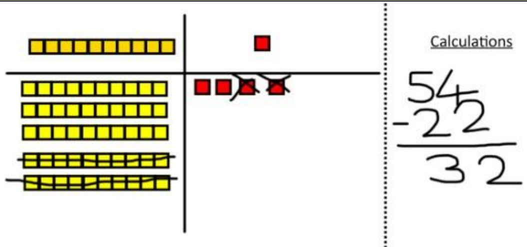
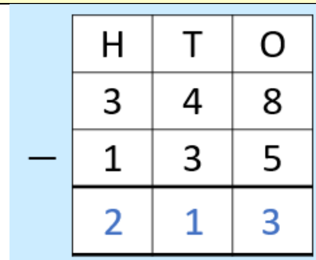
Represent and use number bonds and related subtraction facts within 20 Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what is the other part? 10—6 = 4 Use of Tens frame, number lines and number tracks.  	 Use pictorial representations to show the part.   	 Move to using numbers within the part whole model. When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?

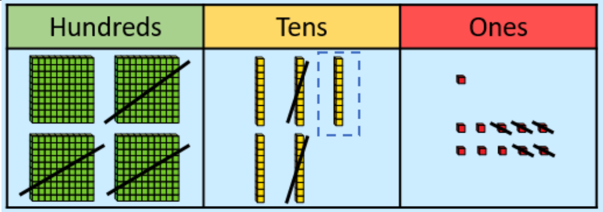
Bar Model	 5 – 2 = 3		<table><tr><td>8</td><td>2</td></tr><tr><td colspan="2">10</td></tr></table> 10 = 8 + 2 10 = 2 + 8 10 – 2 = 8 10 – 8 = 2

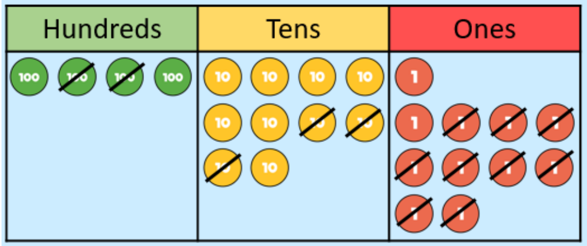
Year 2 Subtraction

Objective and Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 The children can draw dienes and cross them off.	20 – 4 = 16
Partitioning to subtract without regrouping. 'Friendly numbers'	 34—13 = 21 Use Dienes or pv counters to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22

Make ten strategy ***Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.***	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17
Column method	Use dienes or place value counters with a place value grid.  	 	 

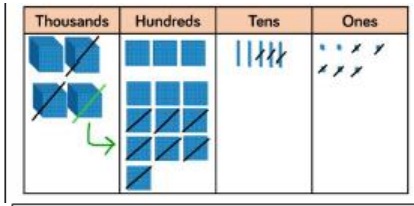
Year 3 Subtraction			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Column subtraction without regrouping (friendly numbers)	  	 Draw representations to support under-standing, Dienes or place value counters.	

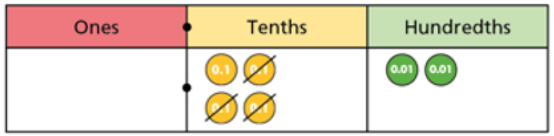
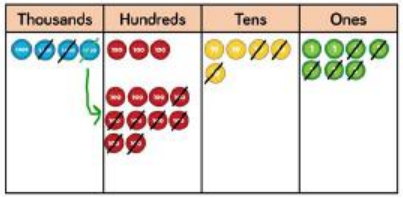
<table>
<tr>
<td>Column subtraction with regrouping</td>
<td>


</td>
<td>The children can draw a representation of the dienes or place vaue counters.</td>
<td>
<table>
<tr><td></td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td>4</td><td>4 ~~5~~</td><td>1 1</td></tr>
<tr><td>–</td><td>3</td><td>2</td><td>5</td></tr>
<tr><td></td><td>1</td><td>2</td><td>6</td></tr>
</table>
<table>
<tr><td></td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td>3 ~~4~~</td><td>9 ~~10~~</td><td>1 1</td></tr>
<tr><td>–</td><td>2</td><td>3</td><td>9</td></tr>
<tr><td></td><td>1</td><td>6</td><td>2</td></tr>
</table>
</td>
</tr>
</table>

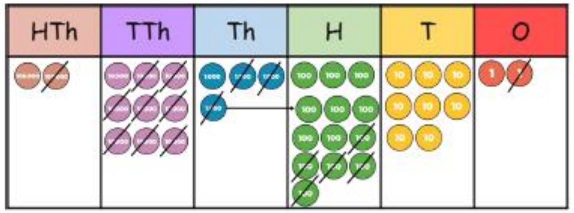
<table>
<tr><th colspan="4"><h1>Year 4 to Year 6 Subtraction</h1></th></tr>
<tr><th>Objective and Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Subtracting tens and ones

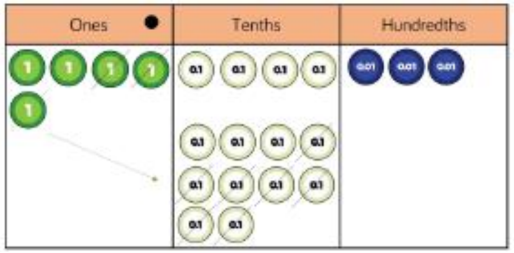
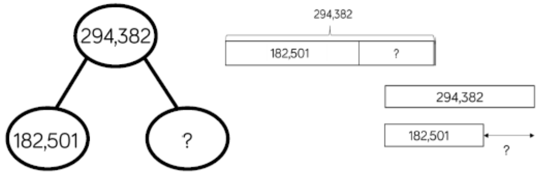
Year 4 subtract with up to 4 digits.

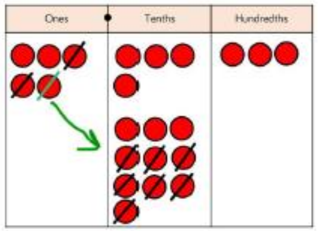
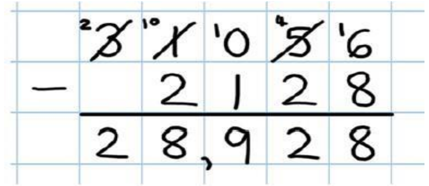
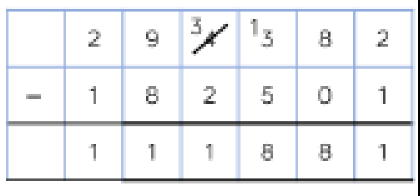
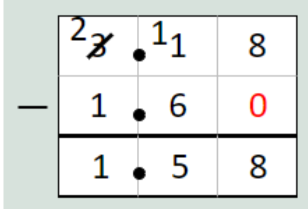
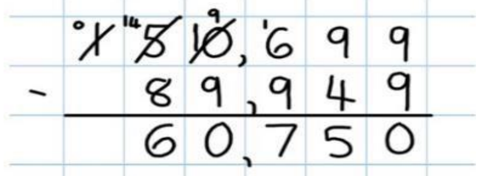
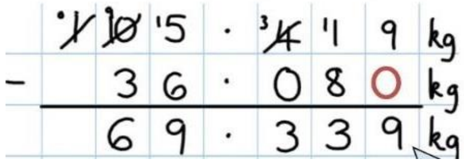
Introduce decimal subtraction through context of money</td>
<td>Model process of exchange using base ten or PV counters.
4,357 – 2,735 = 1,622


a) 0.42 – 0.3 =

</td>
<td>Children to draw pv counters and show their exchange—see Y3

</td>
<td>
$$\begin{array}{rcccc} & 2 & \overset{6}{\cancel{7}} & {}^{1}5 & 4 \\ - & 1 & 5 & 6 & 2 \\ \hline & 1 & 1 & 9 & 2 \end{array}$$
$$\begin{array}{r} \overset{3}{\cancel{4}}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \\ \hline \end{array}$$
Ensure the children write out the column method alongside the concrete resources.</td>
</tr>
</table>

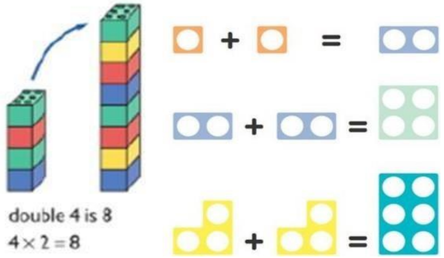
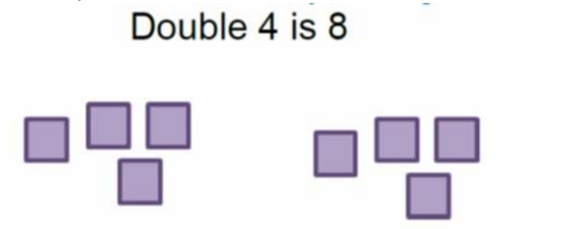
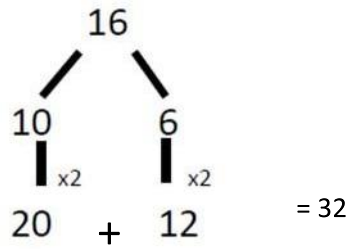
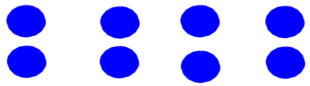
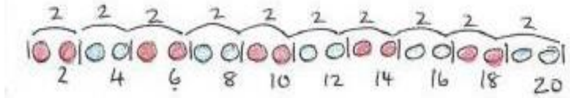
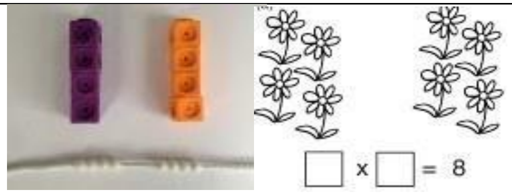
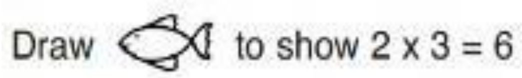
<table>
<tr>
<td>Year 5- Subtract with at least 4 digits, including money and measures.

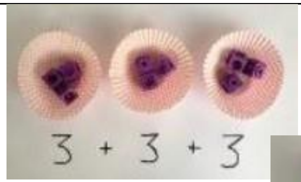
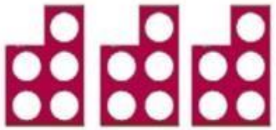
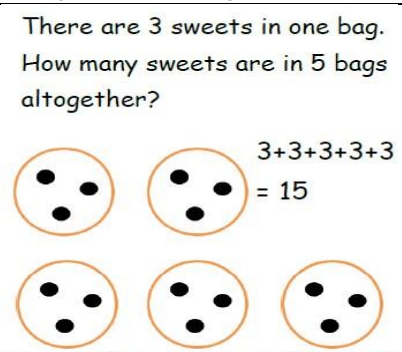
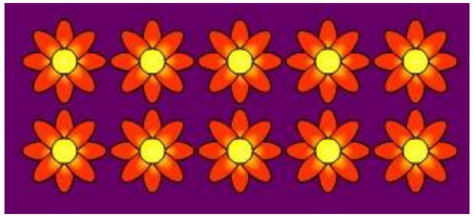
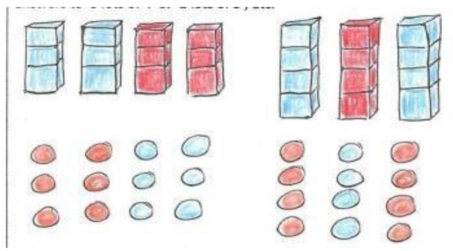
Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</td>
<td>294,382 − 182,501 = 111,881


5.43 − 2.7 = 2.73

</td>
<td>

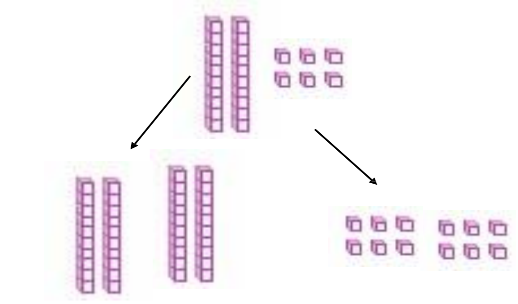
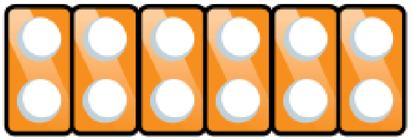
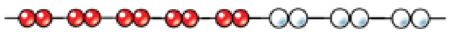
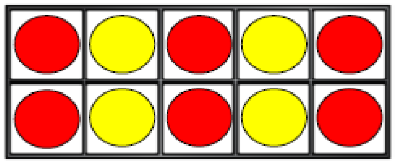
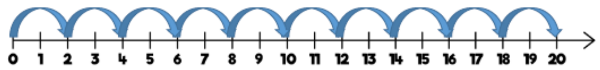
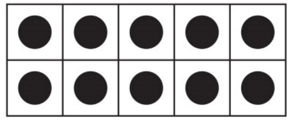
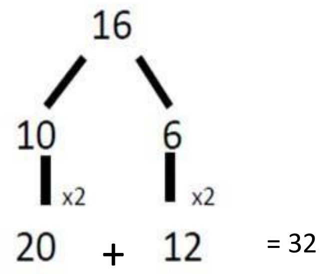
Children to draw pv counters and show their exchange—see Y3

</td>
<td>
Use zeros for place- holders.

</td>
</tr>
<tr>
<td>Year 6—Subtract with increasingly large and more complex numbers and decimal values.</td>
<td></td>
<td></td>
<td>
</td>
</tr>
</table>

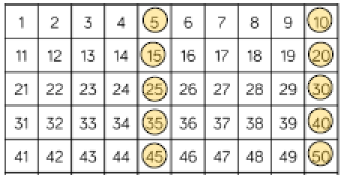
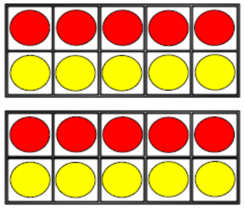
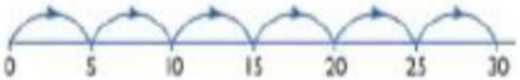
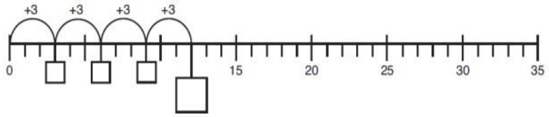
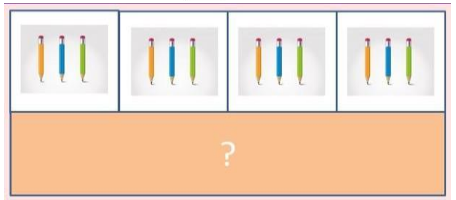
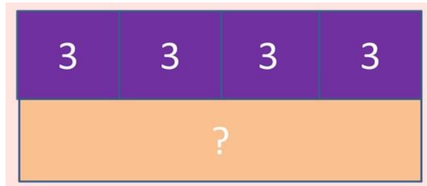
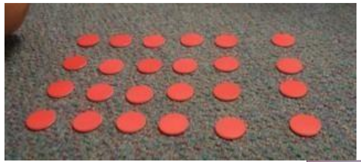
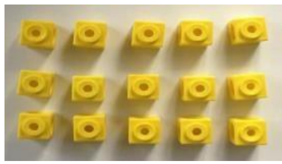
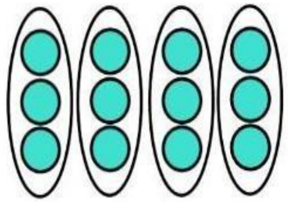
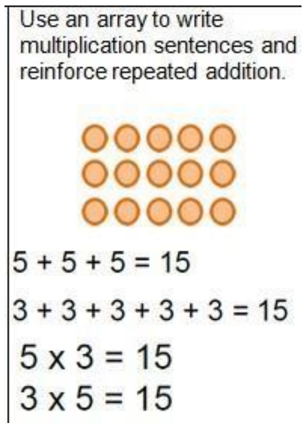
Year 1 Multiplication

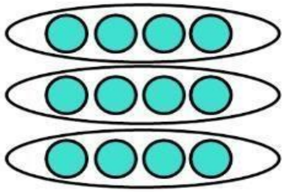
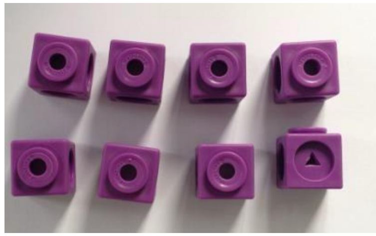
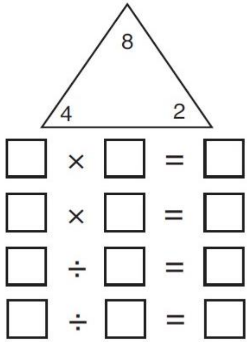
Objective and Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	 Use manipulatives to create equal groups.	 Draw and make representations	2 x 4 = 8

Repeated addition	   Use different objects to add equal groups	  Use pictorial including numberline to solve problems	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

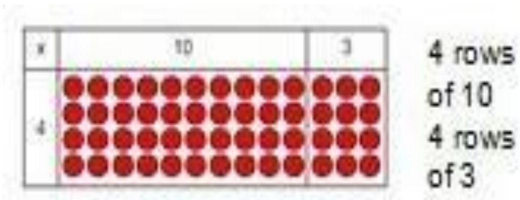
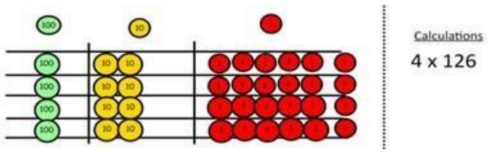
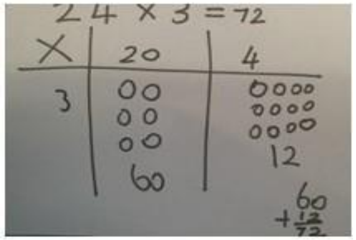
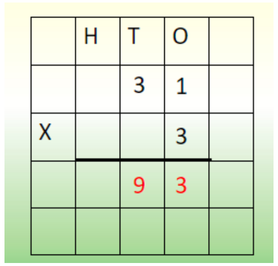
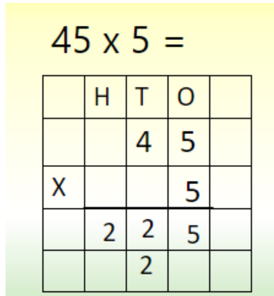
Year 2 Multiplication

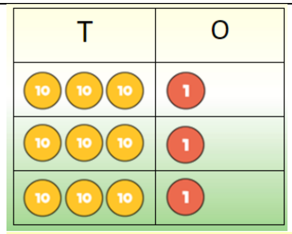
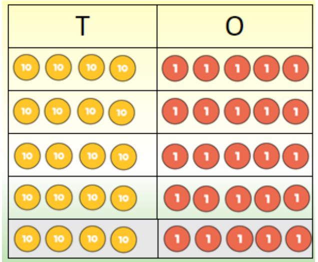
Objective and Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52    	Draw pictures and representations to show how to double numbers  	 Partition a number and then double each part before recombining it back together.

Counting in multiples of 2, 3, 4, 5, 10 from 0 **(repeated addition)**	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40    	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.     	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$ 
Multiplication is commutative	Create arrays using counters and cubes and Numicon.   	Use representations of arrays to show different calculations and explore commutativity. 	12 = 3 × 4  12 = 4 × 3

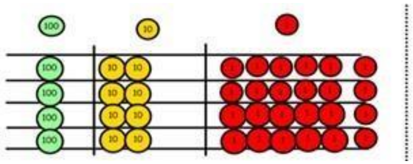
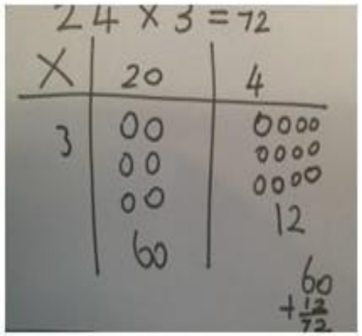
	Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.		
Using the Inverse ***This should be taught alongside division, so pupils learn how they work alongside each other.***			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

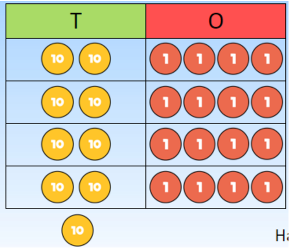
Year 3 Multiplication

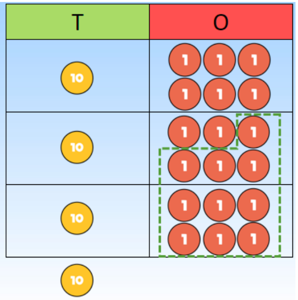
Objective and Strategy	Concrete	Pictorial	Abstract
Grid Method **Column Method**	 Show the links with arrays to first introduce the grid method. Use base ten or place value counters 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	Start with multiplying by 1 digit numbers and showing the clear addition alongside the grid. **×** | **30** | **5** **7** | **210** | **35** **210 + 35 = 245** 

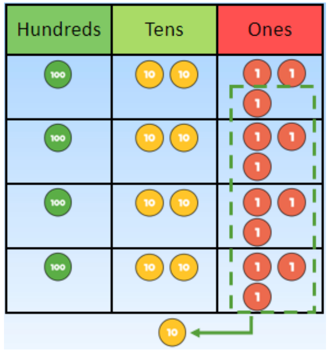
	 		

Year 4 Multiplication

Objective and Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Fill each row with 126	Children can represent their work with place value counters in a way that they understand. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. **×** | **30** | **5** **7** | **210** | **35** **210 + 35 = 245**

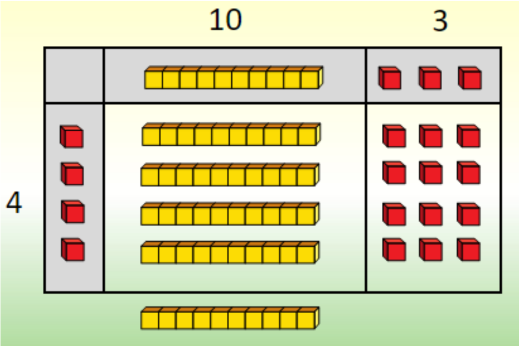
<table>
<tr>
<td>Column multiplication</td>
<td>Use dienes or place value counters to represent the multiplication.


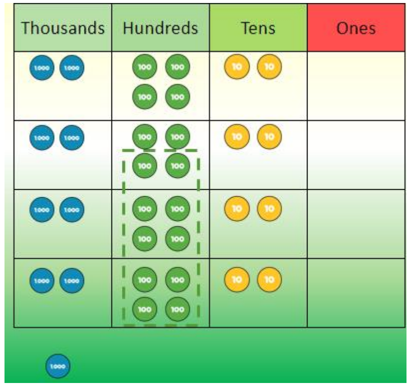




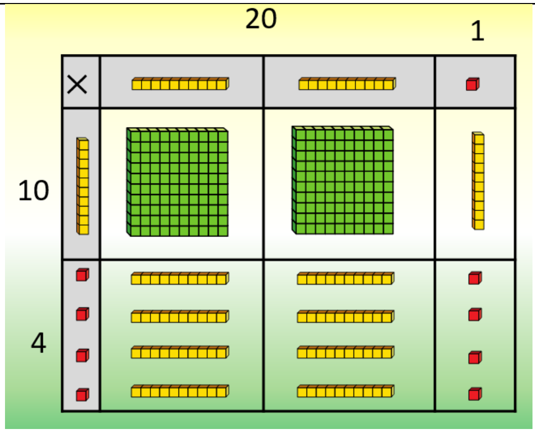
</td>
<td></td>
<td>
<table>
<tr><td></td><td>T</td><td>O</td><td></td></tr>
<tr><td></td><td>2</td><td>4</td><td></td></tr>
<tr><td>×</td><td></td><td>4</td><td></td></tr>
<tr><td></td><td>1</td><td>6</td><td>(4 × 4)</td></tr>
<tr><td></td><td>8</td><td>0</td><td>(4 × 20)</td></tr>
<tr><td></td><td>9</td><td>6</td><td></td></tr>
</table>
<table>
<tr><td></td><td>T</td><td>O</td></tr>
<tr><td></td><td>1</td><td>6</td></tr>
<tr><td>×</td><td></td><td>3</td></tr>
<tr><td></td><td>4</td><td>8</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>
<table>
<tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr>
<tr><td></td><td>1</td><td>2</td><td>3</td><td></td></tr>
<tr><td>×</td><td></td><td></td><td>4</td><td></td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>(3 × 4)</td></tr>
<tr><td></td><td></td><td>8</td><td>0</td><td>(20 × 4)</td></tr>
<tr><td>+</td><td>4</td><td>0</td><td>0</td><td>(100 × 4)</td></tr>
<tr><td></td><td>4</td><td>9</td><td>2</td><td></td></tr>
</table>
<table>
<tr><td></td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>×</td><td></td><td></td><td>4</td></tr>
<tr><td></td><td>4</td><td>9</td><td>2</td></tr>
<tr><td></td><td></td><td>1</td><td></td></tr>
</table>
</td>
</tr>
</table>

Year 5 and Year 6 Multiplication

<table>
<tr><th>Objective and Strategy</th><th>Concrete - Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Long multiplication
The children need to be able to use the grid method and the long column method.</td>
<td>Dienes or place value counters can be used in a place value grid. The children can draw dienes or place value counters. The abstract should be written alongside.




</td>
<td>
<table>
<tr><th>x</th><th>10</th><th>3</th></tr>
<tr><td>4</td><td>40</td><td>12</td></tr>
</table>
40 +
<u>12</u>
<u>52</u>
<table>
<tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr>
<tr><td></td><td>2</td><td>4</td><td>2</td><td>0</td></tr>
<tr><td>×</td><td></td><td></td><td></td><td>4</td></tr>
<tr><td></td><td>9</td><td>6</td><td>8</td><td>0</td></tr>
<tr><td></td><td>1</td><td></td><td></td><td></td></tr>
</table>
</td>
</tr>
</table>



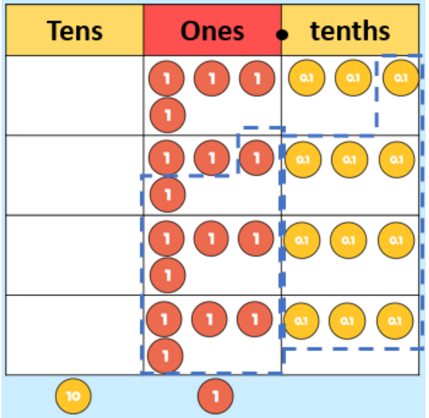
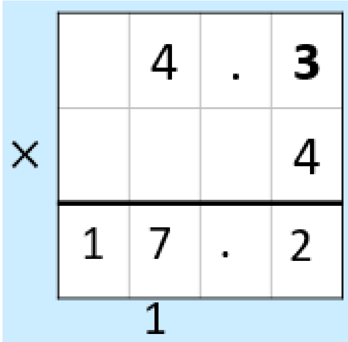
×	20	3
30	600	90
1	20	3

600 + 90 + 20 + 3 = 713

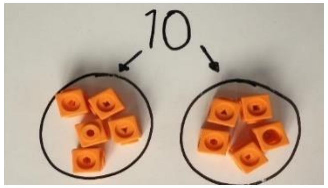
	H	T	O
		2	3
×		3	1
		2	3
+	6	9	0
	7	1	3
	1		

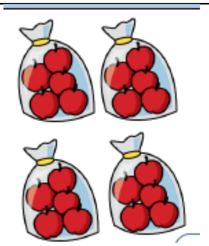
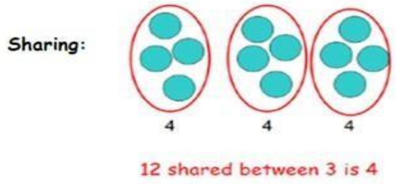
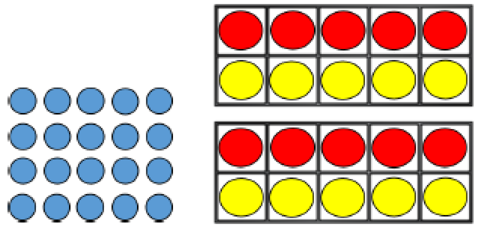
	TTh	Th	H	T	O	
		2	3	1	3	
×				3	2	
		4	6	2	6	(2,313 × 2)
+	6	9	3	9	0	(2,313 × 30)
	7	4	0	1	6	
	1	1	1			

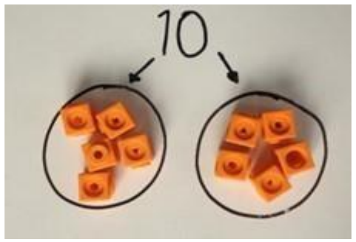
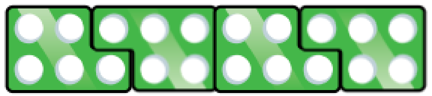
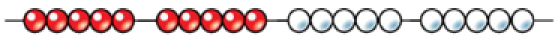
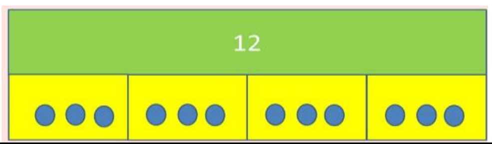
Year 6 Multiplication

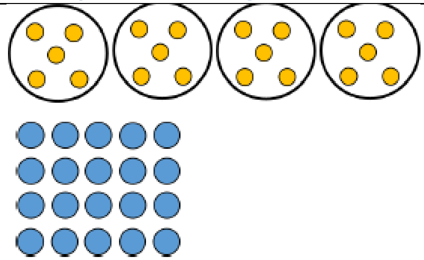
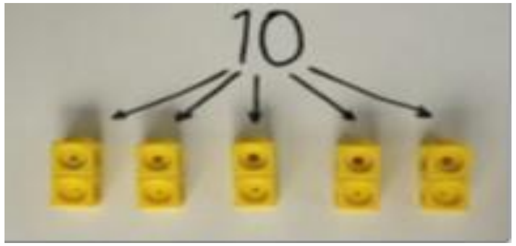
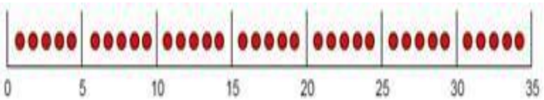
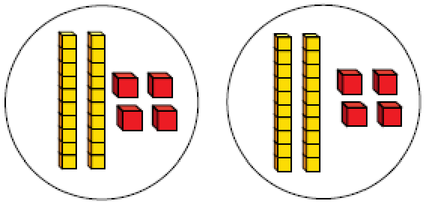
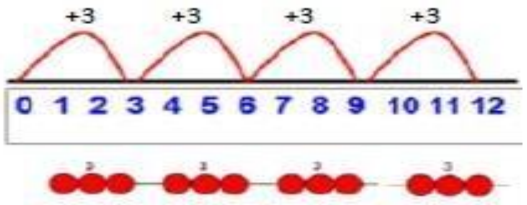
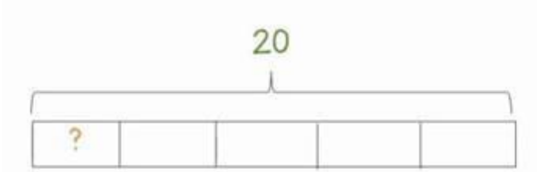
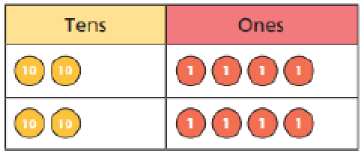
Objective and Strategy	Concrete - Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.	Use place value counters and a place value grid to represent the multiplication. Children can draw the place value grid and place value counters. 	

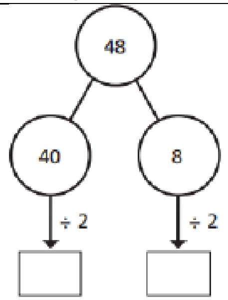
Year 1 Division

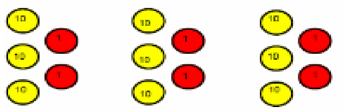
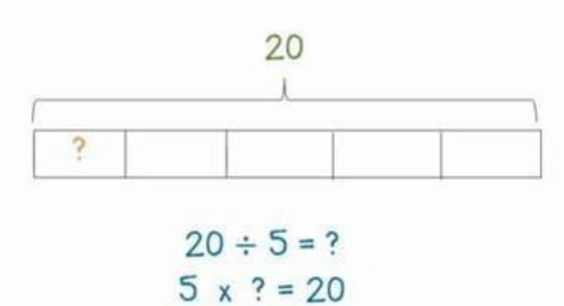
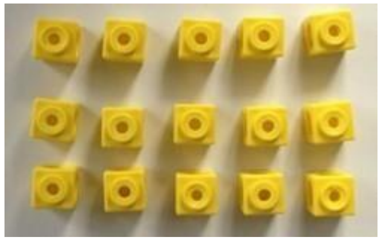
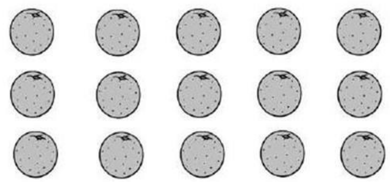
Objective and Strategy	Concrete	Pictorial	Abstract
Division as sharing	  	Children use pictures or shapes to share quantities.  8 shared between 2 is 4	12 shared between 3 is 4

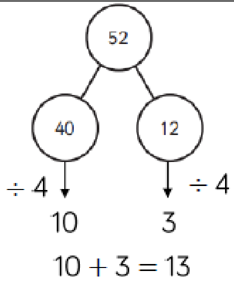
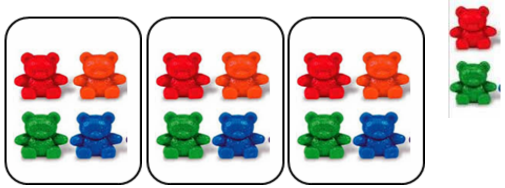
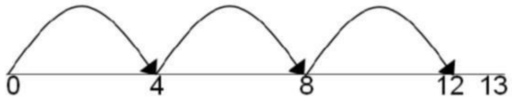
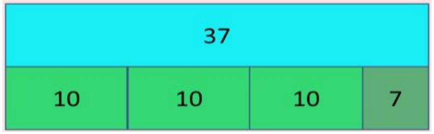
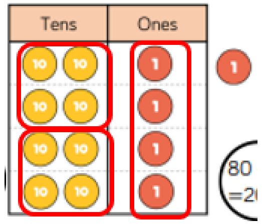
		 	

Year 2 Division			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as sharing	  	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding. 	12 ÷ 3 = 4

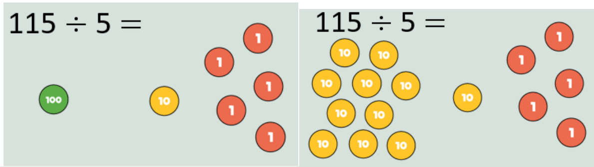
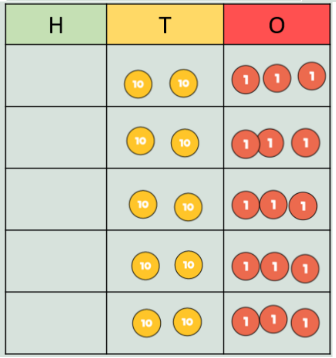
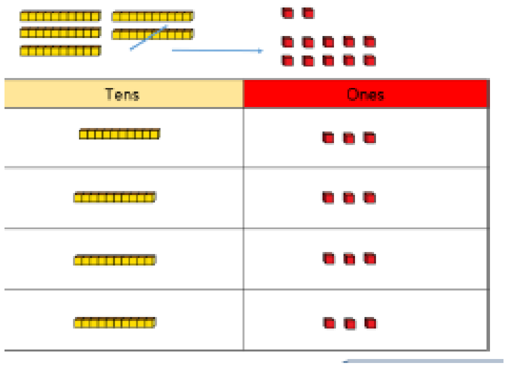
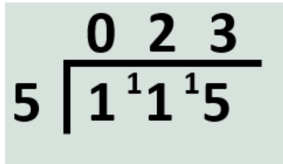
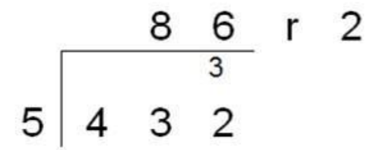
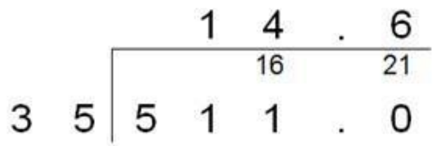
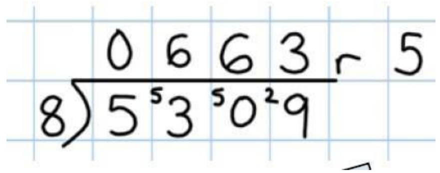
			
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   	Use number lines for grouping  $12 \div 3 = 4$  $20 \div 5 = ?$ $5 \times ? = 20$ Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group? $48 \div 2 = 24$

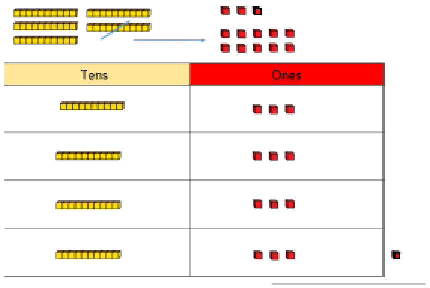
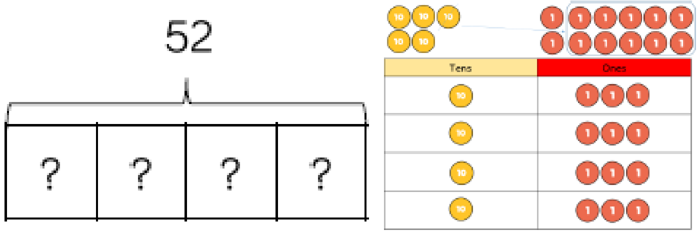
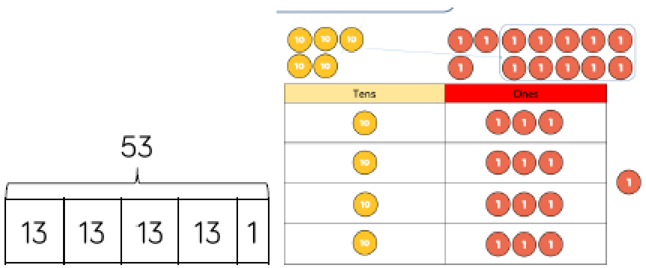
			

Year 3 Division			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays		Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4

	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15		28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders 	Complete written divisions and show the remainder using r. 
Short division Using the bus stop method	 	The children can draw the place value counters and circle the groups for division.	$\begin{array}{l\|ll} & 2 & 2 \\ \hline 3 & 6 & 6 \end{array}$ $\begin{array}{l\|ll} & 2 & 1\ r1 \\ \hline 4 & 8 & 5 \end{array}$

Year 4 to Year 6 Division

Objective and Strategy	Concrete - Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	Children can use dienes or place value counters to divide numbers. Children can drawn diagrams with dots or circles to help them divide numbers into equal groups.   	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

	Tens Ones 52 ? ? ? ? Tens Ones 53 13 13 13 13 1 Tens Ones	